Grade 10 Mathematics

CBC NOTES | Terms 1, 2 and 3 - notes for 14 topics

IN EVERY TOPIC

- What you will learn, the key words and key terms explained.
- Clear notes under headings and subheadings, with labelled pictures and step-by-step procedures.
- Comparison tables, and advantages and disadvantages where methods are compared.
- Key points to remember, then structured questions to practise.
- All the answers and a glossary at the back.

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Strand 1.0 Numbers and Algebra | Term 1

1.1 Real Numbers

WHAT YOU WILL LEARN

By the end of this topic you should be able to:

- classify whole numbers as odd, even, prime and composite in different situations
- classify real numbers as rational and irrational in different situations
- determine the reciprocal of real numbers by division
- determine the reciprocal of real numbers by use of mathematical tables and calculators
- apply reciprocal of real numbers in mathematical computations
- promote the use of real numbers in day-to-day activities

KEY WORDS

prime number, composite number, rational number, irrational number, recurring decimal, real numbers, reciprocal, factor, integer, terminating.

Numbers are everywhere: in prices, in measurements, in M-Pesa codes and in the size of a circle. In this topic we sort whole numbers into odd, even, prime and composite, we sort real numbers into rational and irrational, and we use reciprocals to divide by fractions and to work with rates.

1. Odd, even, prime and composite numbers

Even number
A whole number divisible by 2. It ends in 0, 2, 4, 6 or 8.

Odd number
A whole number that is not divisible by 2. It ends in 1, 3, 5, 7 or 9.

Prime number
A whole number with exactly two different factors, 1 and itself. Examples: 2, 3, 5, 7, 11, 13.

Composite number
A whole number with more than two factors. Examples: 4, 9, 15, 51.

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Classifying whole numbers

Number	Factors	Odd or even	Prime or composite
1	1	Odd	Neither (only one factor)
2	1, 2	Even	Prime (the only even prime)
9	1, 3, 9	Odd	Composite
13	1, 13	Odd	Prime
51	1, 3, 17, 51	Odd	Composite

WATCH OUT
Not every odd number is prime. 51 looks prime but 51 = 3 × 17. The number 1 is neither prime nor composite.

2. Testing for primes and prime factors

Divisibility rules

Divisible by	Rule	Example
2	Last digit is even	346
3	Sum of the digits is divisible by 3	471: 4 + 7 + 1 = 12
5	Last digit is 0 or 5	985
9	Sum of the digits is divisible by 9	738: 7 + 3 + 8 = 18
10	Last digit is 0	2,450

Testing whether n is prime

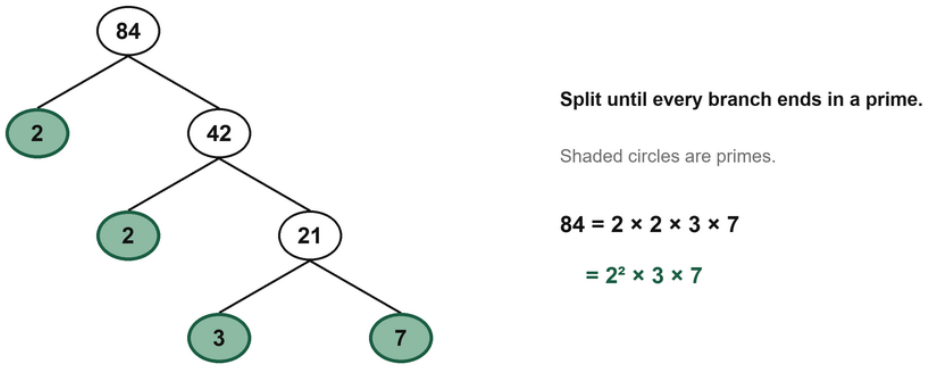
1. Find the primes up to sqrt(n).
2. Divide n by each of these primes.
3. If none divides exactly, n is prime. If one does, n is composite.

Is 221 prime?

sqrt(221) is about 14.9, so test 2, 3, 5, 7, 11 and 13.

221 is odd; 2 + 2 + 1 = 5, so not 3; it does not end in 0 or 5; 221 ÷ 7 and 221 ÷ 11 are not whole numbers.

221 ÷ 13 = 17, so 221 = 13 × 17 and it is composite.



A factor tree for 84

HCF and LCM from prime factors

24 = $2^3 \times 3$ and 36 = $2^2 \times 3^2$.

HCF: take the lowest power of each common prime: $2^2 \times 3 = 12$.

LCM: take the highest power of every prime: $2^3 \times 3^2 = 72$.

Check: 12 × 72 = 864 and 24 × 36 = 864.

3. Rational and irrational numbers

Rational number

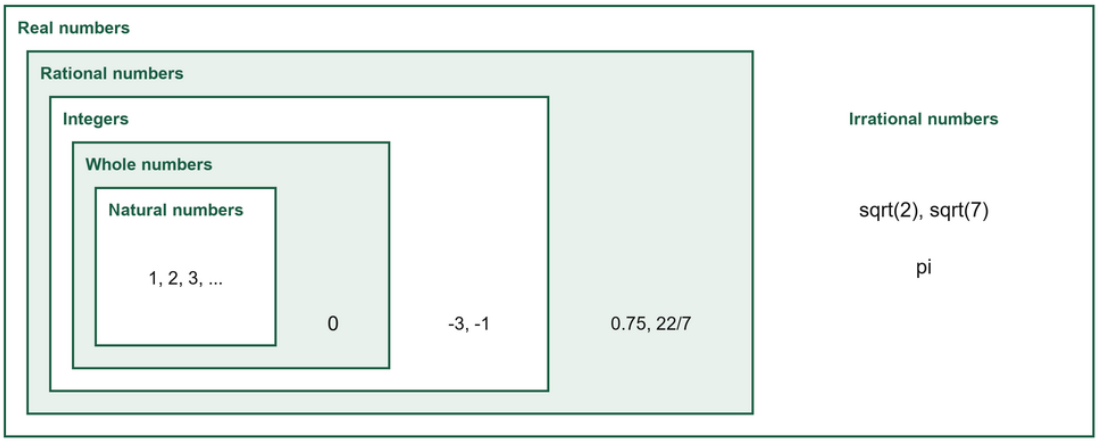
A number that can be written as a/b, where a and b are integers and b is not 0. Integers, fractions, terminating decimals and recurring decimals are rational.

Irrational number

A number that cannot be written as a fraction of two integers. Its decimal never ends and never repeats: sqrt(2), sqrt(5), pi.

Real numbers

All the rational and irrational numbers together. Every point on the number line is a real number.



The sets of numbers fit inside each other

Rational or irrational?

Feature	Rational	Irrational
Written as a/b	Yes	No
Decimal form	Terminates (0.25) or recurs (0.333...)	Never ends and never repeats
Examples	-3, 7, 3/8, 0.75, sqrt(16) = 4	sqrt(2), sqrt(7), sqrt(10), pi

DID YOU KNOW?

22/7 and 3.14 are rational numbers that are only close to pi. Pi itself is irrational.

4. Changing a recurring decimal into a fraction

Method

1. Let x be the recurring decimal.
2. Multiply by 10 if one digit repeats, by 100 if two digits repeat.
3. Subtract to remove the repeating part.
4. Solve for x and simplify the fraction.

Write 0.4545... as a fraction

Let x = 0.4545...

100x = 45.4545...

100x - x = 45, so 99x = 45.

x = 45/99 = 5/11. Check: 5 ÷ 11 = 0.4545...

Write 0.1666... (only the 6 repeats) as a fraction

Let x = 0.1666... Then 10x = 1.666... and 100x = 16.666...

100x - 10x = 15, so 90x = 15.

x = 15/90 = 1/6.

5. Reciprocals

Reciprocal

The reciprocal of x is 1/x, the number which multiplied by x gives 1. 0 has no reciprocal.

Finding reciprocals

Number	Reciprocal	Check
4	¼ = 0.25	4 × 0.25 = 1
2/3	3/2	2/3 × 3/2 = 1
1½ = 3/2	2/3	3/2 × 2/3 = 1
0.25	4	0.25 × 4 = 1
8	0.125	8 × 0.125 = 1

Using tables and calculators

- Calculator: key in the number and press the x^-1 (or 1/x) key.
- Tables: write the number in standard form A × 10^n, read the reciprocal of A, then change the sign of the power of 10.
- Example: 45.6 = 4.56 × 10. The table gives 0.2193 for 4.56, so the reciprocal of 45.6 is 0.2193 × 10^-1 = 0.02193.

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Two taps filling a tank

Tap A fills a tank in 4 hours, so it fills ¼ of the tank each hour. Tap B takes 6 hours, so it fills 1/6 each hour.

Together: ¼ + 1/6 = 3/12 + 2/12 = 5/12 of the tank each hour.

Time for the whole tank is the reciprocal: 12/5 hours = 2.4 hours = 2 hours 24 minutes.

KEY FACT

Dividing by a number is the same as multiplying by its reciprocal: ¾ ÷ 5/8 = ¾ × 8/5 = 6/5.

KEY POINTS TO REMEMBER

- A prime has exactly two factors; a composite has more than two; 1 is neither, and 2 is the only even prime.
- Test n for primes by dividing by the primes up to sqrt(n).
- Rational numbers can be written as a/b; their decimals terminate or recur. Irrational numbers cannot.
- The real numbers are the rational and irrational numbers together.
- The reciprocal of x is 1/x (x not 0); dividing by x is multiplying by 1/x.

ANSWER THESE

1. Express 360 as a product of its prime factors in index form. (2 marks)
2. Use prime factors to find the HCF and the LCM of 48 and 180. (3 marks)
3. Show that 127 is a prime number. (2 marks)
4. Write (a) 0.2727... and (b) 0.8333... (only the 3 repeats) as fractions in their simplest form. (4 marks)
5. Classify each number as rational or irrational: 0.6, sqrt(36), sqrt(18), 3/7, pi, -4. (3 marks)
6. Without a calculator, evaluate 1/0.25 + 1/0.5. (2 marks)
7. Pipe A fills a tank in 6 hours and pipe B in 3 hours. A drain C empties the full tank in 4 hours. If all three are open, how long does it take to fill the empty tank? (3 marks)
8. Work out 2¼ ÷ 3/8. (2 marks)

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Strand 1.0 Numbers and Algebra | Term 1

1.2 Indices and Logarithms

WHAT YOU WILL LEARN

By the end of this topic you should be able to:

- express numbers in index form
- derive the laws of indices using factors
- apply the laws of indices in different situations
- relate index notation to logarithm notation to base 10
- determine common logarithms of numbers from mathematical tables and calculators
- apply common logarithms in multiplication, division, powers and roots of numbers

KEY WORDS

index, base, power, logarithm, antilogarithm, characteristic, mantissa, standard form, root, exponent.

Scientists write the distance to the sun, the size of a bacterium and the growth of savings using powers. Indices are a short way of writing repeated multiplication, and logarithms are indices used to turn hard multiplication, division, powers and roots into easier addition, subtraction, multiplication and division.

1. Index notation

Base and index

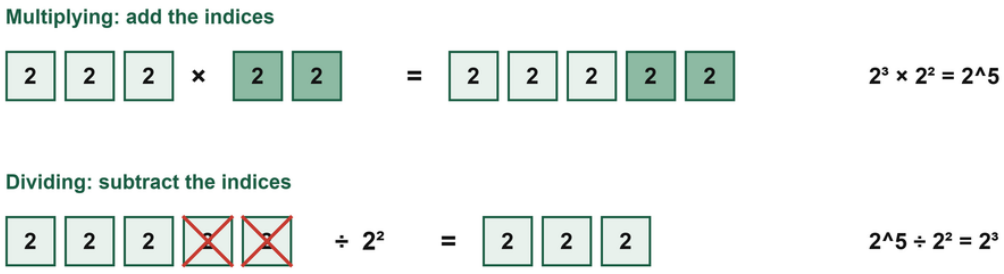
In 2^5 the base is 2 and the index (power) is 5. It means 2 × 2 × 2 × 2 × 2 = 32 and is read "2 to the power of 5".

- 3 × 3 × 3 × 3 = 3^4 = 81.
- 10 × 10 × 10 = 10^3 = 1,000.
- 81 can be written as 3^4 or as 9^2.

COMMON MISTAKE

2^3 means 2 × 2 × 2 = 8, not 2 × 3 = 6.

2. The laws of indices



The laws work only when the bases are the same.

Why the indices add when multiplying and subtract when dividing